

# Berry phase in a non-isolated system

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We investigate the effect of the environment on a Berry phase measurement involving a spin-half. We model the spin+environment using a biased spin-boson Hamiltonian with a time-dependent magnetic field. We find that, contrary to naive expectations, the Berry phase acquired by the spin can be observed, but only on timescales which are neither too short nor very long. However this Berry phase is not the same as for the isolated spin-half. It does not have a simple geometric interpretation in terms of the adiabatic evolution of either bare spin-states or the dressed spin-resonances that remain once we have traced out the environment. This result is crucial for proposed Berry phase measurements in superconducting nanocircuits as dissipation there is known to be significant.

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It was recently suggested [1] that it should be possible to observe the Berry phase (BP)[2] in a superconducting nanostructure, and possibly use it to control the evolution of the quantum state [3, 4]. This intriguing suggestion however did not consider the coupling to the environment, which is never negligible in such structures [5]. To truly understand the feasibility of the proposed experiment, we must know the effect of the environment on the BP. Originally the BP was defined for systems whose states were separated by finite energy gaps. Here we ask whether a BP can be observed in a system whose spectrum is continuous because it is not completely isolated from its environment. All real systems are coupled, at least weakly, to their environment and as a result never have a truly discrete energy level spectrum. The usual requirement for adiabaticity is that the parameters of the Hamiltonian are varied slowly compared to the gap in the spectrum. Here there is no gap so *naively* one would say that adiabaticity is impossible and hence the BP could never be observed. However experiments have observed the BP, both directly and indirectly [6], so this argument must be too naive. We therefore take a simple model in which a quantum system, which when isolated exhibits a BP, is coupled to many other quantum degrees of freedom. We then ask two questions. Firstly, under what conditions can the BP be observed? Secondly, is the observed BP the same as that of the isolated system? While others have investigated systems with a BP coupled to other degrees of freedom [7, 8, 9], we believe we are the first to explicitly address these two questions.

We distinguish between the system and the environment in the following way. We have complete experimental control over the *system*, but almost no control over the *environment*. The most that we can do to the environment is to ensure the “universe” (system + environment) is in thermal equilibrium, with a temperature  $T$ . We will assume we have enough control over  $T$  to take it to zero, and thus prepare the universe in its ground state. However any procedure to measure a BP in an isolated

system must involve measuring a phase difference from a superposition of two states. When the system is not isolated most such procedures involve the mixing of a large number of eigenstates (of the universe), this leads to the effects that we discuss below[10].

We choose to investigate a spin-half which is coupled to both a magnetic field and an environment (a bath of harmonic oscillators). Our model is a biased spin-boson model [11] with a time-dependent field. When isolated from its environment, the spin exhibits a BP if we slowly rotate the magnetic field around a closed loop. This model, chosen primarily for its simplicity, is extremely relevant to a recent proposal for observing a BP in a superconducting nanocircuit [1]. While we make no attempt to accurately model the true coupling between the nanocircuit and its environment, we believe our results give an excellent indication of what to expect in the real system. Our work will also be very relevant to realisations of the BP quantum computers proposed in [3].

In this Letter we concentrate on an *Ohmic environment* [11], with the universe initially at *zero-temperature* [12]. We find that the spin-environment coupling causes the spin-eigenstates to become spin-resonances which have the following properties. (i) The energy distance between them is Lamb shifted by  $\delta E$ . (ii) The higher energy resonance exponentially decays to the lower one on a time-scale,  $T_1$ , and observables containing phase information exponential decay on a timescale  $T_2$ . (iii) There are adiabatic phase-shifts, which divide into two categories with different symmetries; the phase which vanishes when the Hamiltonian is time-independent we call  $\delta\Phi_{BP}$ ; while those phase-shifts (and amplitudes) which do not vanish we schematically refer to as  $\Phi_{shift}$ . The former scales with the winding number of the BP experiment, while the latter does not (see below). All of these effects go like the second power of the spin-environment coupling, see eqs. (4)-(6).

Effect (ii) means that one cannot perform an arbitrarily long experiment to measure a phase: so we must find

the BP from an experiment where the system's Hamiltonian is taken round a closed loop in a finite time period,  $t_p \lesssim T_2$ . In such an experiment there is typically a non-zero amplitude for returning to the initial state and this amplitude has a phase. We interpret the latter as the sum of a dynamic phase which scales linearly with  $t_p$ , an adiabatic phase ( $\Phi_{BP} + \Phi_{shift}$ ) which is independent of  $t_p$ , and non-adiabatic contributions which are proportional to  $\Delta \cdot t_p$  to some negative power[13]. Here  $\Delta$  is the energy difference between the spin-resonances (we set  $\hbar = 1$ ). Thus the BP is present for arbitrary  $t_p$ , it is simply masked by the non-adiabatic contributions unless  $t_p$  is long enough. For the BP to be observed we must choose a value for  $t_p$  which is neither too short nor very long, so that it obeys  $\Delta^{-1} \ll t_p \lesssim T_2$ . However we then actually observe a combination of  $\Phi_{BP}$  and  $\Phi_{shift}$ . To distinguish between these two effects we note that when we do not rotate the Hamiltonian  $\Phi_{BP} = 0$  while  $\Phi_{shift}$  is unchanged.

Now we ask if the environment's effect on the BP is observable. To do this we must first decide what BP we would naively expect to observe. There are two possible cases to consider: (i) The system evolves in a magnetic field that we directly control, then we would expect the BP to be given by the solid-angle enclosed by that field,  $\Phi_{BP}^{(0)}$ . The deviation from this expectation is given by  $\delta\Phi_{BP}$  in eq. (5). For this deviation to be observable it must be much larger than the non-adiabatic corrections at  $t_p \lesssim T_2$ ; this means that  $\Delta \cdot T_2 \cdot \delta\Phi_{BP} \gg 1$ . The functional form of  $T_2$  and  $\delta\Phi_{BP}$ , in (4) and (5), have the same dependence on the strength of the coupling to the environment,  $C$ . Thus the condition reduces to one dominated by the dependence on  $\gamma$ , where  $\gamma$  (defined below eq. (5)) characterises the environment. We conclude that there is a wide range of values of  $\gamma$  for which we can observe  $\delta\Phi_{BP}$ . (ii) The second case is more complicated, but is relevant to the superconducting nanocircuit in [1]. There we have no independent measure of the bare spin Hamiltonian, the control parameters (gate voltages and magnetic fluxes) enter the spin Hamiltonian in combination with unknown constants (capacitances and inductances). Thus we know nothing about the bare spin-eigenstates, or the solid angle that they enclose when we vary the experimental parameters. However we can measure the spin resonances in the presence of the environment as a function of the experimental parameters. Then one might predict the observed BP is given by the solid-angle enclosed by these spin-resonances. This prediction is given above (7); it is of a similar form to the correct result, but contains a very different function of the distribution of oscillators in the environment. The deviation from this expectation is given by  $\delta\Phi'_{BP}$ , for it to be observable we require that  $\Delta \cdot T_2 \cdot \delta\Phi'_{BP} \gg 1$ . Again this reduces to a function independent of  $C$ , where  $\delta\Phi'_{BP}$  is observable over a wide range of  $\gamma$ . Finally, we assume we measure  $\Phi_{shift}$  when for a time-independent Hamilto-

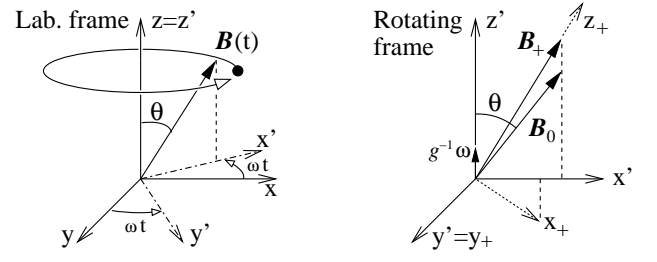


FIG. 1: Evolution in step (b) of the experiment, in Lab. and rotating frames. The primed-basis and plus-basis are both shown.

nian before carrying out the BP experiment. Then we do not require  $\delta\Phi_{BP}$  (or  $\delta\Phi'_{BP}$ ) to be larger than  $\Phi_{shift}$  for it to be observable.

To be concrete we assume here that the BP is measured using the spin-echo method[3]. We consider an experiment where we start with the field along the  $z$ -axis and the universe (spin+oscillators) is in its ground state [14]. (a) The field is then (instantaneously) prepared at its initial value,  $\mathbf{B}_0$ , which is at angle  $\theta$  to the  $z$ -axis. At the same time the spin is (instantaneously) placed in the state  $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$  relative to  $\mathbf{B}_0$ . Then (b) we adiabatically rotate the magnetic field,  $\mathbf{B}(t)$ ,  $n$  times around a closed loop with constant angular velocity,  $\omega = \hat{z}\omega$ , (see Fig. 1) for a time period,  $t_p = 2\pi n/\omega$ . We call  $n$  the winding number. After which (c) the spin is flipped and (d) the field is rotated with angular velocity  $-\omega$  for time  $t_p$ . Finally (e) the spin is flipped again and (f) the spin state is measured. By “flip the spin” we mean  $|\uparrow\rangle \leftrightarrow |\downarrow\rangle$ , where the  $\uparrow$  and  $\downarrow$  are relative to the direction of the  $\mathbf{B}$ -field at that time. This can be achieved by applying a instantaneous  $\pi$ -pulse oriented along the  $y$ -axis. By instantaneous we mean much faster than the fastest oscillator in the environment. We ask what the probability is that the final spin-state, after carrying out (a)-(f), is in a given direction in the plane perpendicular to  $\mathbf{B}_0$ . For an isolated spin-half, the probability of the final spin-state being  $\frac{1}{\sqrt{2}}(e^{i\xi/2}|\uparrow\rangle + e^{-i\xi/2}|\downarrow\rangle)$  is [3]

$$P(\xi) = \frac{1}{2} \left[ 1 + \cos \left( \xi - 4\Phi_{BP}^{(0)} \right) \right], \quad (1)$$

where all spin-states are defined relative to the axis of the field  $\mathbf{B}_0$ . Measuring this probability as a function of  $\xi$  yields the BP for an isolated spin,  $\Phi_{BP}^{(0)} = \pi n(1 - \cos\theta)$ . We wish to know what we observe if we carry out the same measurement for a spin which has been weakly coupled to a bath of oscillators throughout the experiment.

The Hamiltonian we consider contains the spin-half in the above time-dependent magnetic field,  $\mathbf{B}(t)$ , which is also coupled to a bath of harmonic oscillators with frequencies  $\{\Omega_j\}$ . Writing it in terms of creation,  $\hat{b}^\dagger$ , and

annihilation,  $\hat{b}$ , operators for the oscillators,

$$\mathcal{H}(t) = -\frac{g}{2}\mathbf{B}(t) \cdot \hat{\boldsymbol{\sigma}} + \sum_{j,\alpha} \Omega_j \left( \hat{b}_{j,\alpha}^\dagger \hat{b}_{j,\alpha} + \frac{1}{2} \right) - \frac{g}{2} \sum_{j,\alpha} \frac{C_\alpha}{(2m\Omega_j)^{1/2}} \left( \hat{b}_{j,\alpha}^\dagger + \hat{b}_{j,\alpha} \right) \hat{\sigma}_\alpha, \quad (2)$$

where  $j$  is summed over all oscillators and  $\alpha$  is summed over the  $(x, y, z)$  components of the oscillator. The number of oscillators with frequency  $\Omega$  to  $\Omega + d\Omega$  is  $p(\Omega)d\Omega$ . The spectral density [11] is given by  $J(\Omega) = \sum_j \pi (gC)^2 (2m\Omega_j)^{-1} \delta(\Omega - \Omega_j) = \pi (gC)^2 p(\Omega) (2m\Omega)^{-1}$ . Here we restrict ourselves to  $z$ -axis spin-environment coupling with  $C_\alpha = C\delta_{\alpha z}$  [15]. Then for  $\mathbf{B} \cdot \boldsymbol{\sigma} = B\sigma_z$ , the exact ground state of the universe [14] is simply  $|\uparrow\rangle \prod_j |0_j^z\rangle$  where oscillator  $j$  is in the ground state,  $|0_j^z\rangle$ , of the harmonic potential centred at  $(0, 0, \frac{1}{2}gC)$ . We consider an Ohmic bath of oscillators with  $J(\Omega) = \frac{\pi}{2}\tilde{C}^2\Omega \exp[-\Omega/\Omega_m]$ , and work in the limit of small dimensionless coupling  $\tilde{C} \equiv gC(\alpha/m)^{1/2} \ll 1$ .

The time-dependence in (2) makes the problem unpleasant, however we remove this by going to the primed-basis which rotates with the  $\mathbf{B}$ -field. In this non-inertial basis the spin experiences an effective field  $(\mathbf{B}_0 + g^{-1}\boldsymbol{\omega})$ . For our problem the effective field is  $\mathbf{B}_+$  for  $0 < t < t_p$  (shown in Fig. 1), and  $\mathbf{B}_-$  for  $t_p < t < 2t_p$ , where  $\mathbf{B}_\pm = (\mathbf{B}_0 \pm g^{-1}\boldsymbol{\omega}\hat{z})$ . Having removed the time-dependent, we calculate the evolution of the system in the frame which has its  $z$ -axis parallel to the field (either  $\mathbf{B}_+$  or  $\mathbf{B}_-$ ) these frames we call the *plus*- and *minus*-basis respectively (the former is shown in Fig. 1). Finally we rotate back to the lab-frame to evaluate observables.

Before we give a detailed explanation of how we calculate the spin's evolution in the presence of the environment, we giving our results. The anisotropic nature of the coupling results in  $P(\xi)$  containing  $\mathcal{O}[\tilde{C}^2]$ -terms which go like  $\exp[\pm igBt_p]$ . To simplify the resulting expressions we average  $t_p$  over a range  $\gtrsim (gB)^{-1}$  to remove these terms, then

$$P(\xi) = \frac{1}{2} \left[ 1 + e^{-2t_p/T_2} \cos(\xi - 4\Phi_{\text{BP}} - \kappa_1) + |\kappa_2| e^{-2t_p/T_2} \cos(\xi + 4\Phi_{\text{BP}} - \arg[\kappa_2]) + |\kappa_3| (2e^{-2t_p/T_2} - e^{-4t_p/T_2}) \cos(\xi - \arg[\kappa_3]) - \kappa_4 \cos \xi \right], \quad (3)$$

where  $\Phi_{\text{BP}} = \Phi_{\text{BP}}^{(0)} + \delta\Phi_{\text{BP}}$ . For compactness we have dropped an uninteresting real  $\mathcal{O}[\tilde{C}^2 t_p^0]$  term from the first exponent while retaining such terms elsewhere. The  $\kappa$ s (which were schematically referred to as  $\Phi_{\text{shift}}$  above) are  $\mathcal{O}[\tilde{C}^2]$  and so are comparable to  $\delta\Phi_{\text{BP}}$ ; however they are independent of the time-dependence of  $\mathcal{H}(t)$ , and hence independent of the winding number,  $n$ . We find,

$$T_2^{-1} = (2T_1)^{-1} = \frac{\pi}{8}\tilde{C}^2\Omega_m\gamma e^{-\gamma} \sin^2\theta \quad (4)$$

$$\delta\Phi_{\text{BP}} = \frac{\pi}{8}n\tilde{C}^2 [f'(\gamma) - 2\gamma^{-1}f(\gamma)] \sin^2\theta \cos\theta, \quad (5)$$

where  $\gamma = gB/\Omega_m$ . The function  $f(x) = xe^x\text{Ei}(-x) + xe^{-x}\text{Ei}(x)$  where we define  $\text{Ei}(x)$  as the principal-value of the Exponential integral,  $\int_{-x}^\infty dt e^{-t}/t$ , and  $f'(x) \equiv df(x)/dx$ . Eq. (5) is simply the  $\omega$ -dependent term in the Lamb shift of the energy when in the rotating frame. This generates a term of  $\mathcal{O}[n \cdot t_p^0]$  in the phase which in the laboratory frame is a contribution to the BP.

The  $n$ -independent factors are

$$\begin{aligned} \kappa_1 &= \frac{1}{4}\tilde{C}^2\pi e^{-\gamma} \sin\theta \left[ \cos\theta + \frac{1}{4}\sin\theta \right] \\ \kappa_2 &= \frac{1}{16}\tilde{C}^2 (\gamma^{-1}f(\gamma) + i\pi e^{-\gamma}) \sin^2\theta \\ \kappa_3 &= \frac{1}{4}\tilde{C}^2 [(\gamma^{-1}f(\gamma) + i\pi e^{-\gamma}) \sin\theta \cos\theta - 2\gamma^{-1}\sin\theta] \\ \kappa_4 &= \frac{1}{2}\tilde{C}^2 [e^\gamma\text{Ei}(-\gamma) \sin\theta \cos\theta - \gamma^{-1}\sin\theta] . \end{aligned} \quad (6)$$

Now we check that the BP is not simply given by the solid-angle enclosed by the spin-resonances. If this were the case then the BP for this experiment would be  $\Phi_{\text{BP}}^{(0)} - \frac{\pi}{4}n\tilde{C}^2\gamma^{-1}f(\gamma)\sin^2\theta\cos\theta$ , the correct result deviates from this prediction by

$$\delta\Phi'_{\text{BP}} = \frac{\pi}{8}n\tilde{C}^2 f'(\gamma) \sin^2\theta \cos\theta, \quad (7)$$

for most  $\gamma$  this deviation is significant.

We now discuss the method we use to obtain these results. The Hamiltonian in the primed-basis is time-independent and is given by

$$\mathcal{H}'(\mathbf{B}_\pm) = -\frac{g}{2}\mathbf{B}_\pm \cdot \hat{\boldsymbol{\sigma}} + \sum_j \Omega_j \left( \hat{b}_j^\dagger \hat{b}_j + \frac{1}{2} \right) - \frac{g}{2} \sum_j \frac{C}{(2m\Omega_j)^{1/2}} \left( \hat{b}_j^\dagger + \hat{b}_j \right) \hat{\sigma}_z. \quad (8)$$

If we write the spin's initial density matrix as  $\rho_0$  and the oscillators initial density matrix as  $\rho_0^{\text{osc}}$ , then we are interested in the spin density matrix at time  $t$ , after we have traced over the oscillator states,  $\rho_t = \text{tr}_{\text{osc}} \hat{U}_t (\rho_0 \otimes \rho_0^{\text{osc}}) \hat{U}_t^\dagger$ , where  $\hat{U}_t$  is the evolution operator. We find it helpful to write the spin density matrix as a vector  $\boldsymbol{\rho}$  whose elements are  $(\rho_{11}, \rho_{12}, \rho_{21}, \rho_{22})$ . Then the spin evolution equation (after the oscillators have been traced over) can be written as  $\boldsymbol{\rho}_t = \mathbf{K}(t)\boldsymbol{\rho}_0$ , where this defines  $\mathbf{K}(t)$  as a four-by-four matrix which gives the time evolution of the elements of the spin's density matrix. The initial state of the oscillators enters in the functional form of the elements of  $\mathbf{K}(t)$ . For the experiment described above eq. (1) we need to calculate  $\boldsymbol{\rho}_{2t_p} = \mathbf{K}^{\text{flip}} \mathbf{K}(\mathbf{B}_+, t_p) \mathbf{K}^{\text{flip}} \mathbf{K}(\mathbf{B}_-, t_p) \boldsymbol{\rho}_0$ . The spin-flip is assumed to be fast enough to leave all the oscillators unchanged while flipping the spin, then in the primed-basis  $\mathbf{K}^{\text{flip}}$  simply has "1"s on the off-diagonal and "0"s elsewhere. This leaves the calculation of the propagation matrix  $\mathbf{K}(\mathbf{B}_+, t_p)$ , we can find  $\mathbf{K}(\mathbf{B}_-, t_p)$  by reversing the sign of  $\omega$  throughout. For weak coupling to the bath it is natural to work in the *plus*-basis (see Fig. 1), which has its  $z$ -axis parallel to  $\mathbf{B}_+$ , in this basis  $\mathbf{K}(\mathbf{B}_+, t_p)$  becomes diagonal if  $\tilde{C} \rightarrow 0$ . Finally the coupling between spin and oscillators in the plus-basis

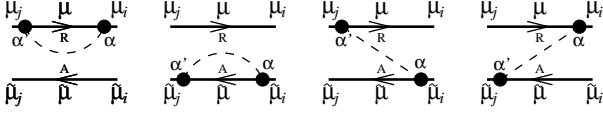


FIG. 2: The four classes of  $\mathcal{O}[\tilde{C}^2]$ -contributions to  $\Sigma^+(\tau)$  are shown here. The upper (lower) line is the Retarded (Advanced) non-interacting ( $C_\alpha = 0$ ) spin-propagator. Everything is in the basis where these propagators are diagonal (the plus-basis for  $\Sigma^+$ ). The spin-state is  $\mu$  with  $\uparrow (\downarrow) \equiv +1(-1)$ . The dotted-lines are the spin-spin interactions after we have traced out the oscillators which couple to the spin via  $\sigma_\alpha$ .

at time  $t$  is  $\mathbf{C}_+(t) = \mathbf{C}'(t)\mathbf{R}_+$ , where  $\mathbf{R}_+$  is the  $\text{SO}(3)$  rotation from the primed-basis to the plus-basis.

Now we use the real-time transport method [16] to write the following differential equation for  $\mathbf{K}^+(t)$ ,

$$\partial_t \mathbf{K}^+(t) = -i\mathbf{E}^+ \mathbf{K}^+(t) + \int_0^t d\tau \Sigma^+(\tau) \mathbf{K}^+(t-\tau), \quad (9)$$

where all bold symbols are  $4 \times 4$  matrices. The matrix  $\mathbf{E}^+$  gives the evolution of the propagation matrix when there is no coupling to the bath. Because we are in the plus-basis it is diagonal with  $E_{11}^+ = E_{44}^+ = 0$  and  $E_{22}^+ = -E_{33}^+ = -gB^+ \equiv -g|\mathbf{B} + g^{-1}\boldsymbol{\omega}|$ . The matrix  $\Sigma^+(\tau)$  is the contribution of all irreducible diagrams with one or more interactions with the bath of oscillators. Equation (9) is exact, however to proceed we treat this equation to first order in  $\tilde{C}^2$ . Thus in the integral on the left hand side of (9) we treat  $\Sigma^+(\tau)$  to first order in  $\tilde{C}^2$  and  $\mathbf{K}^+(t-\tau)$  to zeroth order. So we can write  $\mathbf{K}^+(t-\tau) \simeq \mathbf{K}^+(t)\mathbf{K}_0^+(-\tau)$  where the corrections to the approximations are  $\mathcal{O}[\tilde{C}^2]$  and so can be ignored. Now  $\Sigma^+$ , which is evaluated below, is dominated by small- $\tau$  so we take the upper limit on the integral to infinity. The error we make in doing so is  $\mathcal{O}[(\Omega_m t_p)^{-1}]$  which we neglect. This systematic approximation results in the interaction becoming local in time. Then we get  $\partial_t \mathbf{K}^+(t) = [-i\mathbf{E}^+ + \mathbf{X}^+] \mathbf{K}^+(t)$  where  $\mathbf{X}^+ = \int_0^\infty d\tau \Sigma^+(\tau) \mathbf{K}_0^+(-\tau)$ , and diagonalise the matrix  $(-i\mathbf{E}^+ + \mathbf{X}^+)$  to find  $\mathbf{K}^+(t)$ .

Now we briefly discuss the evaluation of  $\Sigma^+$  to lowest order in  $\tilde{C}^2$ . At this order we need only consider irreducible diagrams with a single interaction with the oscillators. When the oscillators are traced out they leave an interaction between the spin at time  $t$  and time  $t-\tau$ . The resulting first-order irreducible diagrams are shown in Fig. 2. The contribution to  $\Sigma^+$  of the diagram with an interaction via  $\sigma_\alpha$  at time  $t$  and another via  $\sigma_{\alpha'}$  at time  $t'$ , after we have summed over the Ohmic bath, is

$$-\chi \frac{g^2 \alpha}{8m} [\mathbf{C}_+^T(t) \mathbf{C}_+(t')]_{\alpha\alpha'} \times \left\{ \begin{matrix} [\sigma_\alpha]_{\mu_i \mu} \\ [\sigma_\alpha]_{\tilde{\mu} \tilde{\mu}_i} \end{matrix} \right\} \left\{ \begin{matrix} [\sigma_{\alpha'}]_{\mu \mu_j} \\ [\sigma_{\alpha'}]_{\tilde{\mu}_j \tilde{\mu}} \end{matrix} \right\} \frac{\Omega_m^2 e^{igB^+(\mu-\tilde{\mu})\tau/2}}{(1+i\kappa\Omega_m\tau)^2}, \quad (10)$$

where  $\kappa = \pm 1$ ,  $\chi = \pm 1$ , and other variables are shown in

Fig. 2. The upper (lower) term in  $\{\dots\}$  is applicable if the relevant vertex is on R (A).  $\kappa$  is  $+1$  ( $-1$ ) when the  $\alpha'$  vertex is on R (A).  $\chi$  is  $+1$  ( $-1$ ) if the interaction is R-R or A-A (R-A or A-R).

In conclusion, the BP can be observed in a non-isolated system, if the coupling to the environment is weak enough that  $gB \gg T_2^{-1}$ . The adiabatic phase is  $\Phi_{\text{BP}}^{(0)} + \delta\Phi_{\text{BP}} + \Phi_{\text{shift}}$ , but  $\Phi_{\text{shift}}$  is not considered a BP because it does not vanish when  $n = 0$ . So the BP differs from that of a isolated spin by  $\delta\Phi_{\text{BP}}$ , given in Eq. (5). The proportionality of  $\delta\Phi_{\text{BP}}$  to  $n$  hints that it has some geometric character, however it is a function of the environment's spectrum and thus the total BP is not a simple geometric quantity.

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